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Type Package

Title Latent Class Item Response Theory (LC-IRT) Models under Within-Item Multidimensionality

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Description Framework for the Item Response Theory analysis of dichotomous and ordinal polytomous outcomes under the assumption of within-item multidimensionality and discreteness of the latent traits. The fitting algorithms allow for missing responses and for different item parametrizations and are based on the Expectation-Maximization paradigm. Individual covariates affecting the class weights may be included in the new version together with possibility of constraints on all model parameters.

License GPL (>= 2)

Depends R (>= 2.0.0), MASS, limSolve, MultiLCIRT

NeedsCompilation no

Repository https://francescobartolucci.r-universe.dev

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`MLCIRTwithin-package` *Latent Class Item Response Theory (LC-IRT) Models under Within-Item Multidimensionality*

Description

This package provides a flexible framework for the estimation of discrete two-tier Item Response Theory (IRT) models for the analysis of dichotomous and ordinal polytomous item responses. The class of models at issue is based on the assumption that one or more items are shared by (at most) two latent traits (i.e., within-item multidimensionality) and on the discreteness of latent traits (abilities). Every level of the abilities identify a latent class of subjects. The fitting algorithms are based on the Expectation-Maximization (EM) paradigm and allow for missing responses and for different item parametrizations. The package also allows for the inclusion of individual covariates affecting the class weights together with possibility of constraints on all model parameters.

Details

Package:	MultiLCIRT
Type:	Package
Version:	2.1.1
Date:	2019-09-30
License:	GPL (>= 2)

Function `est_multi_poly_within` performs the parameter estimation of the same model considered in the `R` package `MultiLCIRT` when one or more items are shared by two latent traits (within-item multidimensionality); in addition, fixed values and constraints on support points and item parameters are allowed.

Author(s)

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References

Adams, R., Wilson, M., and Wang, W. (1997), The multidimensional random coefficients multinomial logit, *Applied Psychological Measurement*, **21**, 1-24.

Bacci, S. and Bartolucci, F. (2015), A multidimensional finite mixture SEM for non-ignorable missing responses to test items, *Structural Equation Modeling*, **22**, 352-365.

Bacci, S., Bartolucci, F., and Gnaldi, M. (2014), A class of Multidimensional Latent Class IRT models for ordinal polytomous item responses, *Communications in Statistics - Theory and Methods*, **43**, 787-800.

Bartolucci, F. (2007), A class of multidimensional IRT models for testing unidimensionality and clustering items, *Psychometrika*, **72**, 141-157.

Bartolucci, F., Bacci, S., and Gnaldi, M. (2015), *Statistical Analysis of Questionnaires: A Unified Approach Based on R and Stata*, Chapman and Hall/CRC press.

Bartolucci, F., Bacci, S., and Gnaldi, M. (2014), MultiLCIRT: An R package for multidimensional latent class item response models, *Computational Statistics and Data Analysis*, **71**, 971-985.

Bock, R.D., Gibbons, R., and Muraki, E. (1988), Full-information item factor analysis, *Applied Psychological Measurement*, **12**, 261-280.

Bonifay, W. E. (2015), An illustration of the two-tier item factor analysis model, in S. P. Reise and D. A. Revicki (eds), Handbook of Item Response Theory Modeling, p. 207-225, Routledge.

Cai, L. (2010), A two-tier full-information item factor analysis model with applications, *Psychometrika*, **75**, 581-612.

Cai, L., Yang, J. S., and Hansen, M. (2011), Generalized full-information item bifactor analysis, *Psychological Methods*, **16**, 221-248.

Gibbons, R. D., Darrell, R. B., Hedeker, D., Weiss, D. J., Segawa, E., Bhaumik, D. K., and Stover, A. (2007), Full-information item bifactor analysis of graded response data, *Applied Psychological Measurement*, **31**, 4-19.

Gibbons, R. D. and Hedeker, D. R. (1992), Full-information item bi-factor analysis, *Psychometrika*, **57**, 423-436.

Examples

```
## Not run:
# Estimation of a two-tier LC-IRT model
data(SF12_nomiss)
S = SF12_nomiss[,1:12]
X = SF12_nomiss[,13]
```

```
# Define matrices to allocate each item on the latent variables
multi1=rbind(1:6, 7:12)
multi2=rbind(4:8, c(2:3, 10:12))
# Graded response model with two primary latent variables, each of them
# having two dimensions (free discrimination and difficulty parameters;
# two latent classes for both the latent variables; one covariate):
tol = 10^-6 # decrease the tolerance to obtain more reliable results
out1 = est_multi_poly_within(S=S,k1=2,k2=2,X=X,link="global",disc=TRUE,
                             multi1=multi1,multi2=multi2,tol=tol,
                             disp=TRUE,out_se=FALSE)
# Display output
summary(out1)

## End(Not run)
```

`blkdiag` *Build block diagonal matrices*

Description

Function that given two matrices builds the corresponding block diagonal matrix.

Usage

```
blkdiag(A, B)
```

Arguments

`A`	first matrix to be included
`B`	second matrix to be included

Value

`C`	resulting block diagonal matrix

Author(s)

Francesco Bartolucci - University of Perugia (IT)

coef.est_multi_poly_between

Display the estimated model parameters of est_multi_poly_between object

Description

Given the output from est_multi_poly_between, estimated abilities, item parameters, and regression coefficients are displayed

Usage

```
 ## S3 method for class 'est_multi_poly_between'
coef(object, ...)
```

Arguments

object	output from est_multi_poly_between
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

coef.est_multi_poly_within

Display the estimated model parameters of est_multi_poly_within object

Description

Given the output from est_multi_poly_within, estimated abilities, item parameters, and regression coefficients are displayed for the 1st and the 2nd latent variable

Usage

```
 ## S3 method for class 'est_multi_poly_within'
coef(object, ...)
```

Arguments

object	output from est_multi_poly_within
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

`confint.est_multi_poly_between`
Display the estimated confidence intervals of the model parameters of est_multi_poly_between object

---

Description

Given the output from est_multi_poly_between, the inferior and superior limits of confidence intervals at a given level are displayed for abilities, item parameters, and regression coefficients

Usage

```
 ## S3 method for class 'est_multi_poly_between'
confint(object, parm, level=0.95, ...)
```

Arguments

`object`	output from est_multi_poly_between
`parm`	empity object
`level`	confidence level
`...`	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

`confint.est_multi_poly_within`
Display the estimated confidence intervals of the model parameters of est_multi_poly_within object

---

Description

Given the output from est_multi_poly_within, the inferior and superior limits of confidence intervals at a given level are displayed for abilities, item parameters, and regression coefficients for the 1st and the 2nd latent variable

Usage

```
 ## S3 method for class 'est_multi_poly_within'
confint(object, parm, level=0.95, ...)
```

Arguments

object	output from est_multi_poly_within
parm	empity object
level	confidence level
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

---

est_multi_glob_genZ *Fit marginal regression models for categorical responses*

Description

It estimates marginal regression models to datasets consisting of a categorical response and one or more covariates by a Fisher-scoring algorithm; this is an internal function that also works with response variables having a different number of response categories.

Usage

```
est_multi_glob_genZ(Y, X, model = c("m","l","g"), ind = 1:nrow(Y), de = NULL,
                    Z = NULL, z = NULL, Dis = NULL, dis = NULL, disp=FALSE,
                   only_sc = FALSE, Int = NULL, der_single = FALSE, maxit = 10)
```

Arguments

Y	matrix of response configurations
X	array of all distinct covariate configurations
model	type of logit (m = multinomial, l = local, g = global)
ind	vector to link responses to covariates
de	initial vector of regression coefficients
Z	design matrix
z	intercept associated with the design matrix
Dis	matrix for inequality constraints on de
dis	vector for inequality constraints on de
disp	to display partial output
only_sc	to exit giving only the score
Int	matrix of the fixed intercepts
der_single	to require single derivatives
maxit	maximum number of iterations

Value

be	estimated vector of regression coefficients
lk	log-likelihood at convergence
Pdis	matrix of the probabilities for each distinct covariate configuration
P	matrix of the probabilities for each covariate configuration
sc	score for the vector of regression coefficients
FI	Fisher information matrix
de	estimated vector of (free) regression coefficients
scde	score for the vector of (free) regression coefficients
FIde	Fisher information matrix for the vector of (free) regression coefficients
Sc	matrix of individual scores for the vector of regression coefficients (if der_single=TRUE)
Scde	matrix of individual scores for the vector of (free) regression coefficients (if der_single=TRUE)

Author(s)

Francesco Bartolucci - University of Perugia (IT)

References

Colombi, R. and Forcina, A. (2001), Marginal regression models for the analysis of positive association of ordinal response variables, *Biometrika*, **88**, 1007-1019.

Glonek, G. F. V. and McCullagh, P. (1995), Multivariate logistic models, *Journal of the Royal Statistical Society, Series B*, **57**, 533-546.

`est_multi_poly_between`

Estimate latent class item response theory (LC-IRT) models for dichotomous and polytomous responses under between-item multidimensionality

---

Description

The function performs maximum likelihood estimation of the parameters of the IRT models assuming a discrete distribution for the ability and between-item multidimensionality. Every ability level corresponds to a latent class of subjects in the reference population. The class of models is based on a between-item multidimensional formulation with each item loading on a dimension of a given latent variable. Maximum likelihood estimation is based on Expectation- Maximization algorithm.

Usage

```
est_multi_poly_between(S, yv = rep(1, ns), k, X = NULL, start = c("deterministic",
                  "random","external"), link = c("global","local"), disc = FALSE,
                    difl = FALSE, multi = 1:J, Phi = NULL, gat = NULL, De = NULL,
                       fort = FALSE, tol = 10^-10, maxitc = 10^4, disp = FALSE,
                  output = FALSE, out_se = FALSE, glob = FALSE, Zth=NULL,zth=NULL,
                       Zbe=NULL, zbe=NULL,Zga=NULL,zga=NULL)
```

Arguments

S	matrix of all response sequences observed at least once in the sample and listed row-by-row (use NA for missing responses)
yv	vector of the frequencies of every response configuration in S
k	number of ability levels (or latent classes) for the latent variable
X	matrix of covariates that affects the weights
start	method of initialization of the algorithm
link	type of link function ("global" for global logits, "local" for local logits); with global logits a graded response model results; with local logits a partial credit model results (with dichotomous responses, global logits is the same as using local logits resulting in the Rasch or the 2PL model depending on the value assigned to disc)
disc	indicator of constraints on the discriminating indices (FALSE = all equal to one, TRUE = free)
difl	indicator of constraints on the difficulty levels (FALSE = free, TRUE = rating scale parametrization); difl = TRUE is only admitted in the presence of items with the same number of categories
multi	matrix with a number of rows equal to the number of dimensions and elements in each row equal to the indices of the items measuring the dimension corresponding to that row for the latent variable
Phi	initial value of the matrix of the conditional response probabilities (if start="external")
gat	initial value of the vector of free discriminating indices (if start="external")
De	initial value of regression coefficients for the covariates (if start="external")
fort	to use Fortran routines when possible
tol	tolerance level for checking convergence of the algorithm as relative difference between consecutive log-likelihoods
maxitc	maximum number of iterations of the algorithm
disp	to display the likelihood evolution step by step
output	to return additional outputs (Piv,Pp,lkv, Xlabel, XXdis)
out_se	to return standard errors
glob	to use global logits in the covariates
Zth	matrix for the specification of constraints on the support points
zth	vector for the specification of constraints on the support points

Zbe	matrix for the specification of constraints on the item difficulty parameters
zbe	vector for the specification of constraints on the item difficulty parameters
Zga	matrix for the specification of constraints on the item discriminating indices
zga	vector for the specification of constraints on the item discriminating indices

Value

piv	estimated vector of weights of the latent classes (average of the weights in case of model with covariates)
fv	vector indicating the reference item chosen for each latent dimension of the latent variable
tht	estimated matrix of free ability levels for each dimension
Th	complete matrix of free and constrained ability levels for each dimension and latent class of the latent variable
bet	estimated vector of free difficulty levels for every item (split in two vectors if difl=TRUE)
Bec	complete vector of free and constrained difficulty levels for every item (split in two vectors if difl=TRUE)
gat	estimated vector of free discriminating indices for every item (with all elements equal to 1 if disc=FALSE)
gac	complete vector of free and constrained discriminating indices for every item (with all elements equal to 1 if disc=FALSE)
De	matrix of regression coefficients for the multinomial (or global if glob=TRUE) logit model on the class weights
Phi	array of the conditional response probabilities for every item and each of the k latent classes
lk	log-likelihood at convergence of the EM algorithm
np	number of free parameters
aic	Akaike Information Criterion index
bic	Bayesian Information Criterion index
ent	entropy index to measure the separation of classes
pivs	estimated vector of (ordered) weights of the latent classes (average of the weights in case of model with covariates)
Ths	standardized ability levels
Becs	standardized values of item difficulty parameters
gacs	standardized values of item discriminating indices
call	call of function
Pp	matrix of the posterior probabilities for each response configuration and latent class (if output=TRUE)
lkv	vector to trace the log-likelihood evolution across iterations (if output=TRUE)
Xlabel	structure of the design matrix, for internal use (if output=TRUE)

XXdis	design matrix for the covariates affecting the latent variable (if output=TRUE)
Piv	matrix of the weights for every response configuration (if output=TRUE)
setht	standard errors for vector tht (if out_se=TRUE)
seTh	standard errors for vector Th (if out_se=TRUE)
sebet	standard errors for vector bet (if out_se=TRUE)
seBec	standard errors for vector Bec (if out_se=TRUE)
segat	standard errors for vector gat (if out_se=TRUE)
segac	standard errors for vector gac (if out_se=TRUE)
seDe	standard errors for vector De (if out_se=TRUE)
Vnt	estimated variance-covariance matrix for free parameter estimates (if out_se=TRUE)
Vn	estimated variance-covariance matrix for all parameter estimates (if out_se=TRUE)

Author(s)

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References

Bartolucci, F. (2007), A class of multidimensional IRT models for testing unidimensionality and clustering items, *Psychometrika*, **72**, 141-157.

Bacci, S., Bartolucci, F. and Gnaldi, M. (2014), A class of Multidimensional Latent Class IRT models for ordinal polytomous item responses, *Communications in Statistics - Theory and Methods*, **43**, 787-800.

Examples

```
## Not run:
# Fit a Graded response model with two dimensions (free discrimination
# and difficulty parameters; three latent classes):
data(SF12_nomiss)
S = SF12_nomiss[,1:12]
X = SF12_nomiss[,13]
multi0 = rbind(c(1:5, 8), c(6:7,9:12))
k=3

out1 =  est_multi_poly_between(S=S,k=k,X=X,link="global",disc=TRUE,
                               multi=multi0,fort=TRUE,disp=TRUE,out_se=TRUE)

# Display output:
summary(out1)
out1$lk
out1$Th
out1$piv
out1$De

## End(Not run)

## Not run:
```

```
## Fit the model under different external constraints on abilities and/or item parameters
# Fixed ability levels; all item parameters can be free
S1 = pmin(as.matrix(S),2) # all items have the same number of categories
Zth = matrix(0,nrow(multi0)*k,0)
zth = c(rep(-1, times=nrow(multi0)), rep(0, times=nrow(multi0)), rep(1, times=nrow(multi0)))
Zbe = diag(ncol(S1)*2)  # free item difficulties: 12*2  = 24 (12 items with 3 categories)
Zga = diag(ncol(S1));  # free item discriminating parameters = 12 items loading on U
outc1 = est_multi_poly_between(S=S1,k=k,X=X,link="global",disc=TRUE, multi=multi0,disp=TRUE,
                               out_se=TRUE,Zth=Zth,zth=zth,Zbe=Zbe,Zga=Zga)
outc1$Th
outc1$tht
outc1$Bec

# Add equality constraints on item parameters
# Same difficulties for pairs of items 1-7, 2-8, 3-9, 4-10, 5-11, 6-12;
# same discriminating indices for items 2 and 3;
# free ability levels
Zbe = (matrix(1,2,1)%x%diag(12))[,-1]
Zga = as.matrix(rep(0, times=12)); Zga[2,1] = 1; Zga[3,1] = 1;
Zga1p1 = matrix(0, nrow=3, ncol=9); Zga1p2 = diag(9); Zga1p = rbind(Zga1p1, Zga1p2)
Zga = cbind(Zga, Zga1p)
# discriminating index of item 1 constrained to 1 for the model identifiability
zga = rep(0,nrow(Zga)); zga[1] = 1
outc2 = est_multi_poly_between(S=S1,k=k,X=X,link="global",disc=TRUE,
                               multi = multi0,disp=TRUE,tol=10^-4,
                               out_se=TRUE,Zbe=Zbe, Zga=Zga, zga=zga)
outc2$tht
outc2$Th
outc2$Ths
outc2$Bec
outc2$Becs
outc2$gac
outc2$gacs

## End(Not run)
```

`est_multi_poly_within` *Estimate latent class item response theory (LC-IRT) models for dichotomous and polytomous responses under within-item multidimensionality*

Description

The function performs maximum likelihood estimation of the parameters of the two-tier IRT models assuming a discrete distribution for the ability and within-item multidimensionality. Every ability level corresponds to a latent class of subjects in the reference population. The class of models is based on a particular within-item multidimensional formulation with each item loading on at most two uncorrelated latent variables. Maximum likelihood estimation is based on the Expectation-Maximization algorithm.

Usage

```
est_multi_poly_within(S, yv = rep(1, ns), k1, k2, X = NULL,
                start = c("deterministic","random","external"), link = c("global",
                  "local"), disc = FALSE, difl = FALSE, multi1, multi2, Phi = NULL,
                    ga1t = NULL, ga2t = NULL, De1 = NULL, De2 = NULL, fort = FALSE,
                       tol = 10^-10, maxitc = 10^4, disp = FALSE, output = FALSE,
                 out_se = FALSE, glob = FALSE, Zth1 = NULL, zth1 = NULL, Zth2=NULL,
                  zth2=NULL, Zbe=NULL, zbe=NULL, Zga1=NULL, zga1=NULL, Zga2=NULL,
                       zga2=NULL)
```

Arguments

S	matrix of all response sequences observed at least once in the sample and listed row-by-row (use NA for missing responses)
yv	vector of the frequencies of every response configuration in S
k1	number of ability levels (or latent classes) for the 1st latent variable
k2	number of ability levels (or latent classes) for the 2nd latent variable
X	matrix of covariates that affects the weights
start	method of initialization of the algorithm
link	type of link function ("global" for global logits, "local" for local logits); with global logits a graded response model results; with local logits a partial credit model results (with dichotomous responses, global logits is the same as using local logits resulting in the Rasch or the 2PL model depending on the value assigned to disc)
disc	indicator of constraints on the discriminating indices (FALSE = all equal to one, TRUE = free)
difl	indicator of constraints on the difficulty levels (FALSE = free, TRUE = rating scale parametrization); difl = TRUE is only admitted in the presence of items with the same number of categories
multi1	matrix with a number of rows equal to the number of dimensions and elements in each row equal to the indices of the items measuring the dimension corresponding to that row for the 1st latent variable
multi2	matrix with a number of rows equal to the number of dimensions and elements in each row equal to the indices of the items measuring the dimension corresponding to that row for the 2nd latent variable
Phi	initial value of the matrix of the conditional response probabilities (if start="external")
ga1t	initial value of the vector of free discriminating indices (if start="external") for the 1st latent variable
ga2t	initial value of the vector of free discriminating indices (if start="external") for the 2nd latent variable
De1	initial value of regression coefficients for the covariates (if start="external") affecting the 1st latent variable
De2	initial value of regression coefficients for the covariates (if start="external") affecting the 2nd latent variable

fort	to use Fortran routines when possible
tol	tolerance level for checking convergence of the algorithm as relative difference between consecutive log-likelihoods
maxitc	maximum number of iterations of the algorithm
disp	to display the likelihood evolution step by step
output	to return additional outputs (Piv1, Piv2, Pp1, Pp2, lkv, Xlabel, XX1dis, XX2dis)
out_se	to return standard errors
glob	to use global logits in the covariates
Zth1	matrix for the specification of constraints on the support points for the 1st latent variable
zth1	vector for the specification of constraints on the support points for the 1st latent variable
Zth2	matrix for the specification of constraints on the support points for the 2nd latent variable
zth2	vector for the specification of constraints on the support points for the 2nd latent variable
Zbe	matrix for the specification of constraints on the item difficulty parameters
zbe	vector for the specification of constraints on the item difficulty parameters
Zga1	matrix for the specification of constraints on the item discriminating indices for the 1st latent variable
zga1	vector for the specification of constraints on the item discriminating indices for the 1st latent variable
Zga2	matrix for the specification of constraints on the item discriminating indices for the 2nd latent variable
zga2	vector for the specification of constraints on the item discriminating indices for the 2nd latent variable

Details

In order to ensure the model identifiability, the following conditions must hold. First, suitable constraints on the item parameters are required: one discriminanting index must be equal to 1 and one difficulty parameter must be equal to 0 for each dimension. The constrained items may be chosen in an arbitrary way: by default the algorithm selects the first element of each row of `multi1` and `multi2`. As a consequence, the user must pay attention to specify matrices `multi1` and `multi2` so that different items are constrained for each dimension. Second, the maximum number of items shared by the two latent variables is equal to the total number of items minus one, that is, the union of rows of `multi1` must differ from the union of rows of `multi2`. These conditions may be skipped specifying in a suitable way the entries of `Zth1`, `zth1`, `Zth2`, `zth2`, `Zbe`, `zbe`, `Zga1`, `zga1`, `Zga2`, and `zga2`, according to the following equations:

Th1 = Zth1 %*% th1t + zth1

Th2 = Zth2 %*% th2t + zth2

Bec = Zbe %*% bet + zbe

ga1c = Zga1 %*% ga1t + zga1

ga2c = Zga2 %*% ga2t + zga2,

where Th1, Th2, Bec, ga1c, ga2c denote the complete matrices/vectors of support points (Th1, Th2), item difficulties (Bec), and item discriminating indices (ga1c, ga2c), whereas th1t, th2t, bet, ga1t, ga2t are the corresponding matrices/vectors of free (i.e., unconstrained) parameters.

Value

piv1	estimated vector of weights of the latent classes (average of the weights in case of model with covariates) for the 1st latent variable
piv2	estimated vector of weights of the latent classes (average of the weights in case of model with covariates) for the 2nd latent variable
fv1	vector indicating the reference item chosen for each latent dimension for the 1st latent variable
fv2	vector indicating the reference item chosen for each latent dimension for the 2nd latent variable
th1t	estimated matrix of free ability levels for each dimension and for the 1st latent variable
th2t	estimated matrix of free ability levels for each dimension and for the 2nd latent variable
Th1	complete matrix of free and constrained ability levels for each dimension and latent class for the 1st latent variable
Th2	complete matrix of free and constrained ability levels for each dimension and latent class for the 2nd latent variable
bet	estimated vector of free difficulty levels for every item (split in two vectors if difl=TRUE)
Bec	complete vector of free and constrained difficulty levels for every item (split in two vectors if difl=TRUE)
ga1t	estimated vector of free discriminating indices for every item (with all elements equal to 1 if disc=FALSE) for the 1st latent variable
ga2t	estimated vector of free discriminating indices for every item (with all elements equal to 1 if disc=FALSE) for the 2nd latent variable
ga1c	complete vector of free and constrained discriminating indices for every item for the 1st latent variable (with all elements equal to 1 if disc=FALSE and NA for items that do not load on the 1st latent variable)
ga2c	complete vector of free and constrained discriminating indices for every item for the 2nd latent variable (with all elements equal to 1 if disc=FALSE and NA for items that do not load on the 2nd latent variable)
De1	matrix of regression coefficients for the multinomial (or global if glob=TRUE) logit model on the class weights for the 1st latent variable
De2	matrix of regression coefficients for the multinomial (or global if glob=TRUE) logit model on the class weights for the 2nd latent variable
Phi	array of the conditional response probabilities for every item and each of the k1*k2 latent classes

lk	log-likelihood at convergence of the EM algorithm
np	number of free parameters
aic	Akaike Information Criterion index
bic	Bayesian Information Criterion index
ent	entropy index to measure the separation of classes
piv1s	estimated vector of (ordered) weights of the latent classes (average of the weights in case of model with covariates) for the 1st standardized latent variable
piv2s	estimated vector of (ordered) weights of the latent classes (average of the weights in case of model with covariates) for the 2nd standardized latent variable
Th1s	standardized ability levels for the 1st latent variable, ordered according to the first dimension
Th2s	standardized ability levels for the 2nd latent variable, ordered according to the first dimension
Becs	standardized values of item difficulty parameters
ga1cs	standardized values of item discriminating indices for the 1st latent variable
ga2cs	standardized values of item discriminating indices for the 2nd latent variable
call	call of function
Pp1	matrix of the posterior probabilities for each response configuration and latent class for the 1st latent variable (if output=TRUE)
Pp2	matrix of the posterior probabilities for each response configuration and latent class for the 2nd latent variable (if output=TRUE)
lkv	vector to trace the log-likelihood evolution across iterations (if output=TRUE)
Xlabel	structure of the design matrix, for internal use (if output=TRUE)
XX1dis	design matrix for the covariates affecting the 1st latent variable (if output=TRUE)
XX2dis	design matrix for the covariates affecting the 2nd latent variable (if output=TRUE)
Piv1	matrix of the weights for every covariate pattern configuration for the 1st latent variable (if output=TRUE)
Piv2	matrix of the weights for every covariate pattern configuration for the 2nd latent variable (if output=TRUE)
seth1t	standard errors for vector th1t (if out_se=TRUE)
seth2t	standard errors for vector th2t (if out_se=TRUE)
seTh1	standard errors for vector Th1 (if out_se=TRUE)
seTh2	standard errors for vector Th2 (if out_se=TRUE)
sebet	standard errors for vector bet (if out_se=TRUE)
seBec	standard errors for vector Bec (if out_se=TRUE)
sega1t	standard errors for vector ga1t (if out_se=TRUE)
sega2t	standard errors for vector ga2t (if out_se=TRUE)
sega1c	standard errors for vector ga1c (if out_se=TRUE)
sega2c	standard errors for vector ga2c (if out_se=TRUE)
seDe1	standard errors for vector De1 (if out_se=TRUE)
seDe2	standard errors for vector De2 (if out_se=TRUE)
Vnt	estimated variance-covariance matrix for free parameters (if out_se=TRUE)
Vn	complete variance-covariance matrix for all parameters (if out_se=TRUE)

Author(s)

Francesco Bartolucci, Silvia Bacci - University of Perugia (IT)

References

Bacci, S. and Bartolucci, F. (2015), A multidimensional finite mixture SEM for non-ignorable missing responses to test items, *Structural Equation Modeling*, **22**, 352-365.

Bacci, S., Bartolucci, F., and Gnaldi, M. (2014), A class of Multidimensional Latent Class IRT models for ordinal polytomous item responses, *Communications in Statistics - Theory and Methods*, **43**, 787-800.

Bartolucci, F. (2007), A class of multidimensional IRT models for testing unidimensionality and clustering items, *Psychometrika*, **72**, 141-157.

Bonifay, W. E. (2015), An illustration of the two-tier item factor analysis model, in S. P. Reise and D. A. Revicki (eds), Handbook of Item Response Theory Modeling, p. 207-225, Routledge.

Cai, L. (2010), A two-tier full-information item factor analysis model with applications, Psychometrika, 75, 581-612.

Cai, L., Yang, J. S., and Hansen, M. (2011), Generalized full-information item bifactor analysis, Psychological Methods, 16, 221-248.

Examples

```
## Not run:
# Fit the model under different within-item multidimensional structures
# for SF12_nomiss data
data(SF12_nomiss)
S = SF12_nomiss[,1:12]
X = SF12_nomiss[,13]

# Graded response model with two latent variables sharing six items (free
# discrimination and difficulty parameters; two latent classes for each
# latent variable; one covariate):
multi1 = c(1:5, 8:12)
multi2 = c(6:12, 1)
tol = 10^-6  # decrease tolerance to obtain more reliable results
out1 = est_multi_poly_within(S=S,k1=2,k2=2,X=X,link="global",disc=TRUE,
                             multi1=multi1,multi2=multi2,disp=TRUE,
                             out_se=TRUE,tol=tol)

# Partial credit model with two latent variables sharing eleven items
# (free discrimination and difficulty parameters; two latent classes for
# the 1st latent variable and three latent classes for the 2nd latent
# variable; one covariate):
multi1 = 1:12
multi2 = 2:12
out2 = est_multi_poly_within(S=S,k1=2,k2=3,X=X,link="local",disc=TRUE,
                             multi1=multi1,multi2=multi2,disp=TRUE,tol=tol)

# Display output:
summary(out2)
```

```
out2$lk
out2$Th1
out2$Th1s
out2$piv1
out2$Th2
out2$Th2s
out2$piv2
out2$De1
out2$De2

## End(Not run)

## Not run:
## Fit the model under different situations for RLMS data
# Example of use of the function to account for non-ignorable missing
# item responses
data(RLMS)
X = RLMS[,1:4]
Y = RLMS[,6:9]
YR = cbind(Y,1*(!is.na(Y)))
multi1 = 1:4
multi2 = 5:8
tol = 10^-6  # decrease tolerance to obtain more reliable results

# MAR model
out0 = est_multi_poly_within(YR,k1=3,k2=2,X=X,link="global",
                 disc=TRUE,multi1=multi1,multi2=multi2,disp=TRUE,
                 out_se=TRUE,glob=TRUE,tol=tol)

# NMAR model
multi1 = 1:8
out1 = est_multi_poly_within(YR,k1=3,k2=2,X=X,link="global",
                 disc=TRUE,multi1=multi1,multi2=multi2,disp=TRUE,
                 out_se=TRUE,glob=TRUE,tol=tol)

# testing effect of the latent trait on missingness
c(out0$bic,out1$bic)
(test1 = out1$ga1c[-1]/out1$sega1c[-1])

## End(Not run)

## Not run:
## Fit the model under different external constraints on abilities and/or item parameters
data(SF12_nomiss)
S = SF12_nomiss[,1:12]
X = SF12_nomiss[,13]
multi1m = rbind(1:5, 8:12) # two dimensions for the 1st latent variable
multi2m = rbind(6:9, c(10:12, 1)) # two dimensions for the 2nd latent variable
k1 = 2
k2 = 2

# Fixed ability levels; all item parameters can be free
Zth1 = matrix(0,nrow(multi1m)*k1,0)
```

```
zth1 = c(rep(-1, times=nrow(multi1m)),  rep(1, times=nrow(multi1m)))
Zth2 = matrix(0,nrow(multi2m)*k2,0)
zth2 = c(rep(-1, times=nrow(multi2m)),  rep(1, times=nrow(multi2m)))
# item difficulties: 10*4 + 2*2 = 44 (10 items with 5 categories plus 2 items with 3 categories)
Zbe = diag(44)
# item discriminating parameters = 10 items loading on the 1st latent variable plus 8 items loading
# on the 2nd latent variable
Zga1 = diag(10); Zga2 = diag(8)
zga1 = rep(0,nrow(Zga1)); zga1[1] = 1
zga2 = rep(0,nrow(Zga2)); zga2[1] = 1
out1c = est_multi_poly_within(S=S,k1=k1,k2=k2,X=X,link="global",disc=TRUE,multi1=multi1m,
                    multi2=multi2m,disp=TRUE,out_se=TRUE,Zth1=Zth1,zth1=zth1,Zth2=Zth2,
                            zth2=zth2,Zbe=Zbe,Zga1=Zga1,zga1=zga1,Zga2=Zga2,zga2=zga2)
summary(out1c)
out1c$Bec

# Constraint difficulties of the first threshold to be equal for all items
# and difficulties of the second threshold to be equal for all items;
# free ability levels
multi1u = c(1:3, 6:10) # one dimension for the 1st latent variable
multi2u = c(4:10, 1)  # one dimension for the 2nd latent variable
S1 = pmin(as.matrix(S[, -c(2,3)]),2)  # all items have the same number of categories
Zbe = as.matrix((matrix(1,10,1)%x%diag(2))[,-1])
out2c = est_multi_poly_within(S=S1,k1=2,k2=2,X=X,link="global",disc=TRUE,
                            multi1=multi1u,multi2=multi2u,disp=TRUE,
                            out_se=TRUE,Zbe=Zbe)
out2c$Bec

# Same difficulties for pairs of items 1-6, 2-7, 3-8, 4-9, 5-10;
# free ability levels
Zbe = (matrix(1,2,1)%x%diag(10))[,-1]
out3c = est_multi_poly_within(S=S1,k1=2,k2=2,X=X,link="global",disc=TRUE,
                            multi1=multi1u,multi2=multi2u,disp=TRUE,
                            out_se=TRUE,Zbe=Zbe)
out3c$Bec

# Add equality constraints on some discriminating indices for the 1st latent variable
Zbe = (matrix(1,2,1)%x%diag(10))[,-1]
Zga1 = diag(length(multi1u));
# discriminating index of item 1 constrained to 1 for the model identifiability
# discriminating index of item 3 equal to discriminating index of item 2
Zga1 = Zga1[, -c(1, 3)];
Zga1[3, 1] = 1
zga1 = rep(0,nrow(Zga1)); zga1[1] = 1
out4c = est_multi_poly_within(S=S1,k1=2,k2=2,X=X,link="global",disc=TRUE,
                            multi1=multi1u,multi2=multi2u,disp=TRUE,tol=10^-4,
                            out_se=TRUE,Zbe=Zbe, Zga1=Zga1, zga1=zga1)
out4c$Bec
out4c$ga1c
out4c$ga1t

## End(Not run)
```

`lk_obs_score_between` *Compute observed log-likelihood and score*

Description

Function used within `est_multi_poly_between` to compute observed log-likelihood and score.

Usage

```
lk_obs_score_between(part_comp, lde, lpart, lgat, S, R, yv, k, rm, lv,
                     J, fv, disc, glob, refitem, miss,
                    ltype, XXdis, Xlabel, ZZ0,fort, Zpar, zpar, Zga, zga, items)
```

Arguments

`part_comp`	complete vector of parameters
`lde`	length of de
`lpart`	length of part
`lgat`	length of gat
`S`	matrix of responses
`R`	matrix of observed responses indicator
`yv`	vector of frequencies
`k`	number of latent classes for the latent variable
`rm`	number of dimensions for the latent variable
`lv`	number of response categories for each item
`J`	number of items
`fv`	indicator of constrained parameters
`disc`	presence of discrimination parameters
`glob`	indicator of global parametrization for the covariates
`refitem`	vector of reference items
`miss`	indicator of presence of missing responses
`ltype`	type of logit
`XXdis`	array of covariates for the latent variable
`Xlabel`	indicator for covariate configuration
`ZZ0`	design matrix
`fort`	to use Fortran
`Zpar`	array for the specification of constraints on the support points of the latent variable and for the item difficulty parameters
`zpar`	vector for the specification of constraints on the support points of the latent variable and for the item difficulty parameters
`Zga`	matrix for the specification of constraints on the item discriminating indices
`zga`	vector for the specification of constraints on the item discriminating indices
`items`	items affected by the latent variable

Value

lk	log-likelihood function
sc	score vector

Author(s)

Francesco Bartolucci - University of Perugia (IT)

---

lk_obs_score_within *Compute observed log-likelihood and score*

Description

Function used within est_multi_poly_within to compute observed log-likelihood and score.

Usage

```
lk_obs_score_within(part_comp, lde1, lde2, lpart, lga1t, lga2t, S, R, yv, k1, k2,
             rm1, rm2, lv, J, fv, disc, glob, refitem, miss, ltype, XX1dis, XX2dis,
             Xlabel, ZZ0, fort, Zpar, zpar, Zga1, zga1, Zga2, zga2, items1, items2)
```

Arguments

part_comp	complete vector of parameters
lde1	length of de1
lde2	length of de2
lpart	length of part
lga1t	length of ga1t
lga2t	length of ga2t
S	matrix of responses
R	matrix of observed responses indicator
yv	vector of frequencies
k1	number of latent classes for the 1st latent variable
k2	number of latent classes for the 2nd latent variable
rm1	number of dimensions for the 1st latent variable
rm2	number of dimensions for the 2nd latent variable
lv	number of response categories for each item
J	number of items
fv	indicator of constrained parameters
disc	presence of discrimination parameters
glob	indicator of global parametrization for the covariates

refitem	vector of reference items
miss	indicator of presence of missing responses
ltype	type of logit
XX1dis	array of covariates for the 1st latent variable
XX2dis	array of covariates for the 2nd latent variable
Xlabel	indicator for covariate configuration
ZZ0	design matrix
fort	to use Fortran
Zpar	array for the specification of constraints on the support points of the 1st and the 2nd latent variable and for the item difficulty parameters
zpar	vector for the specification of constraints on the support points of the 1st and the 2nd latent variable and for the item difficulty parameters
Zga1	matrix for the specification of constraints on the item discriminating indices for the 1st latent variable
zga1	vector for the specification of constraints on the item discriminating indices for the 1st latent variable
Zga2	matrix for the specification of constraints on the item discriminating indices for the 2nd latent variable
zga2	vector for the specification of constraints on the item discriminating indices for the 2nd latent variable
items1	items affected by the 1st latent variable
items2	items affected by the 2nd latent variable

Value

lk	log-likelihood function
sc	score vector

Author(s)

Francesco Bartolucci - University of Perugia (IT)

---

logLik.est_multi_poly_between

Display the log-likelihood at convergence of est_multi_poly_between object

---

Description

Given the output from est_multi_poly_between, the log-likelihood at convergence is displayed

Usage

```
 ## S3 method for class 'est_multi_poly_between'
logLik(object, ...)
```

Arguments

object	output from est_multi_poly_between
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

---

logLik.est_multi_poly_within

Display the log-likelihood at convergence of est_multi_poly_within object

---

Description

Given the output from est_multi_poly_within, the log-likelihood at convergence is displayed

Usage

```
 ## S3 method for class 'est_multi_poly_within'
logLik(object, ...)
```

Arguments

object	output from est_multi_poly_within
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

`print.est_multi_poly_between` *Print the output of est_multi_poly_between object*

Description

Given the output from est_multi_poly_between, the call of it is written

Usage

```
 ## S3 method for class 'est_multi_poly_between'
print(x, ...)
```

Arguments

`x`	output from est_multi_poly_between
`...`	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

`print.est_multi_poly_within` *Print the call of est_multi_poly_within object*

Description

Given the output from est_multi_poly_within, the call of it is written

Usage

```
 ## S3 method for class 'est_multi_poly_within'
print(x, ...)
```

Arguments

`x`	output from est_multi_poly_within
`...`	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

`prob_multi_glob_gen` *Global probabilities*

Description

It provides matrix of probabilities under different parametrizations and for the case of response variables having a different number of categories.

Usage

```
prob_multi_glob_gen(X, model, be, ind=(1:dim(X)[3]))
```

Arguments

`X`	array of all distinct covariate configurations
`model`	type of logit (g = global, l = local, m = multinomial)
`be`	initial vector of regression coefficients
`ind`	vector to link responses to covariates

Value

`Pdis`	matrix of distinct probability vectors
`P`	matrix of the probabilities for each covariate configuration

Author(s)

Francesco Bartolucci - University of Perugia (IT)

References

Colombi, R. and Forcina, A. (2001), Marginal regression models for the analysis of positive association of ordinal response variables, *Biometrika*, **88**, 1007-1019.

Glonek, G. F. V. and McCullagh, P. (1995), Multivariate logistic models, *Journal of the Royal Statistical Society, Series B*, **57**, 533-546.

RLMS *RLMS dataset*

Description

This dataset contains the data about job satisfaction described in: Bartolucci, F., Bacci, S., and Gnaldi, M. (2015), Statistical Analysis of Questionnaires: A Unified Approach Based on R and Stata, Chapman and Hall/CRC press

Usage

```
data(RLMS)
```

Format

A data frame with 1485 observations about four polytomous items with covariates:

`marital` marital status of the respondent

`education` educational level of the respondent

`gender` gender of the respondent

`age` age of the respondent

`work` work status of the respondent

`Y1` 1st item response

`Y2` 2nd item response

`Y3` 3rd item response

`Y4` 4th item response

References

Bartolucci, F., Bacci, S., and Gnaldi, M. (2015), *Statistical Analysis of Questionnaires: A Unified Approach Based on R and Stata*, Chapman and Hall/CRC press

Examples

```
data(RLMS)
## maybe str(RLMS)
str(RLMS)
```

search.model_between *Search for the global maximum of the log-likelihood of between-item muldimensional models*

Description

It searches for the global maximum of the log-likelihood of between-item muldimensional models given a vector of possible number of classes to try for.

Usage

```
search.model_between(S, yv = rep(1, ns), kv, X = NULL,
                     link = c("global","local"), disc = FALSE, difl = FALSE,
                     multi = 1:J, fort = FALSE, tol1 = 10^-6, tol2 = 10^-10,
                     glob = FALSE, disp = FALSE, output = FALSE,
                     out_se = FALSE, nrep = 2, Zth=NULL,zth=NULL,
                     Zbe=NULL, zbe=NULL,Zga=NULL,zga=NULL)
```

Arguments

S	matrix of all response sequences observed at least once in the sample and listed row-by-row (use NA for missing responses)
yv	vector of the frequencies of every response configuration in S
kv	vector of the possible numbers of latent classes
X	matrix of covariates affecting the weights
link	type of link function ("global" for global logits, "local" for local logits); with global logits a graded response model results; with local logits a partial credit model results (with dichotomous responses, global logits is the same as using local logits resulting in the Rasch or the 2PL model depending on the value assigned to disc)
disc	indicator of constraints on the discriminating indices (FALSE = all equal to one, TRUE = free)
difl	indicator of constraints on the difficulty levels (FALSE = free, TRUE = rating scale parametrization)
multi	matrix with a number of rows equal to the number of dimensions and elements in each row equal to the indices of the items measuring the dimension corresponding to that row for the latent variable
fort	to use Fortran routines when possible
tol1	tolerance level for checking convergence of the algorithm as relative difference between consecutive log-likelihoods (initial check based on random starting values)
tol2	tolerance level for checking convergence of the algorithm as relative difference between consecutive log-likelihoods (final converngece)
glob	to use global logits in the covariates

disp	to display the likelihood evolution step by step
output	to return additional outputs (Piv,Pp,lkv)
out_se	to return standard errors
nrep	number of repetitions of each random initialization
Zth	matrix for the specification of constraints on the support points
zth	vector for the specification of constraints on the support points
Zbe	matrix for the specification of constraints on the item difficulty parameters
zbe	vector for the specification of constraints on the item difficulty parameters
Zga	matrix for the specification of constraints on the item discriminating indices
zga	vector for the specification of constraints on the item discriminating indices

Value

out.single	output of each single model for each k in kv; it is similar to output from est_multi_poly_between, with the addition of values of number of latent classes (k) and the sequence of log-likelihoods (lktrace) for the deterministic start, for each random start, and for the final estimation obtained with a tolerance level equal to tol2
aicv	Akaike Information Criterion index for each k in kv
bicv	Bayesian Information Criterion index for each k in kv
entv	Entropy index for each k in kv
necv	NEC index for each k in kv
lkv	log-likelihood at convergence of the EM algorithm for each k in kv
errv	trace of any errors occurred during the estimation process for each k in kv

Author(s)

Francesco Bartolucci, Silvia Bacci - University of Perugia (IT)

References

Bartolucci, F., Bacci, S. and Gnaldi, M. (2014), MultiLCIRT: An R package for multidimensional latent class item response models, *Computational Statistics & Data Analysis*, **71**, 971-985.

Examples

```
## Not run:
# Fit a Graded response model with two latent variables (free discrimination
# and difficulty parameters; two latent classes):
data(SF12_nomiss)
S = SF12_nomiss[,1:12]
X = SF12_nomiss[,13]
multi0 = rbind(c(1:5, 8), c(6:7,9:12))
out1 = search.model_between(S=S,kv=1:3,X=X,link="global",disc=TRUE,
                                multi=multi0,fort=TRUE,disp=TRUE,out_se=TRUE)

# Display output
```

```
out1$lkv
out1$bicv

# Display output with 2 classes:
out1$out.single[[2]]
out1$out.single[[2]]$lktrace
out1$out.single[[2]]$Th
out1$out.single[[2]]$piv
out1$out.single[[2]]$gac
out1$out.single[[2]]$Bec


## End(Not run)
```

`search.model_within` *Search for the global maximum of the log-likelihood of within-item muldimensional models*

Description

It searches for the global maximum of the log-likelihood of within-item muldimensional models given a vector of possible number of classes to try for.

Usage

```
search.model_within(S, yv = rep(1, ns), kv1, kv2, X = NULL,
                    link = c("global","local"), disc = FALSE, difl = FALSE,
                    multi1, multi2, fort = FALSE, tol1 = 10^-6, tol2 = 10^-10,
                    glob = FALSE, disp = FALSE, output = FALSE, out_se = FALSE,
                    nrep = 2, Zth1 = NULL, zth1 = NULL, Zth2=NULL, zth2=NULL,
                    Zbe=NULL, zbe=NULL, Zga1=NULL, zga1=NULL, Zga2=NULL,
                    zga2=NULL)
```

Arguments

`S`	matrix of all response sequences observed at least once in the sample and listed row-by-row (use NA for missing responses)
`yv`	vector of the frequencies of every response configuration in `S`
`kv1`	vector of the possible numbers of ability levels (or latent classes) for the 1st latent variable
`kv2`	vector of the possible numbers of ability levels (or latent classes) for the 2nd latent variable
`X`	matrix of covariates affecting the weights

link	type of link function ("global" for global logits, "local" for local logits); with global logits a graded response model results; with local logits a partial credit model results (with dichotomous responses, global logits is the same as using local logits resulting in the Rasch or the 2PL model depending on the value assigned to disc)
disc	indicator of constraints on the discriminating indices (FALSE = all equal to one, TRUE = free)
difl	indicator of constraints on the difficulty levels (FALSE = free, TRUE = rating scale parametrization)
multi1	matrix with a number of rows equal to the number of dimensions and elements in each row equal to the indices of the items measuring the dimension corresponding to that row for the 1st latent variable
multi2	matrix with a number of rows equal to the number of dimensions and elements in each row equal to the indices of the items measuring the dimension corresponding to that row for the 2nd latent variable
fort	to use Fortran routines when possible
tol1	tolerance level for checking convergence of the algorithm as relative difference between consecutive log-likelihoods (initial check based on random starting values)
tol2	tolerance level for checking convergence of the algorithm as relative difference between consecutive log-likelihoods (final convergence)
glob	to use global logits in the covariates
disp	to display the likelihood evolution step by step
output	to return additional outputs (Piv,Pp,lkv)
out_se	to return standard errors
nrep	number of repetitions of each random initialization
Zth1	matrix for the specification of constraints on the support points for the 1st latent variable
zth1	vector for the specification of constraints on the support points for the 1st latent variable
Zth2	matrix for the specification of constraints on the support points for the 2nd latent variable
zth2	vector for the specification of constraints on the support points for the 2nd latent variable
Zbe	matrix for the specification of constraints on the item difficulty parameters
zbe	vector for the specification of constraints on the item difficulty parameters
Zga1	matrix for the specification of constraints on the item discriminating indices for the 1st latent variable
zga1	vector for the specification of constraints on the item discriminating indices for the 1st latent variable
Zga2	matrix for the specification of constraints on the item discriminating indices for the 2nd latent variable
zga2	vector for the specification of constraints on the item discriminating indices for the 2nd latent variable

Value

out.single	output of each single model for each k in kv1 and kv2; it is similar to output from est_multi_poly_within, with the addition of values of number of latent classes for the 1st latent variable (k1) and the 2nd latent variable (k2) and the sequence of log-likelihoods (lktrace) for the deterministic start, for each random start, and for the final estimation obtained with a tolerance level equal to tol2
aicv	Akaike Information Criterion index for each k in kv1 and kv2
bicv	Bayesian Information Criterion index for each k in kv1 and kv2
entv	Entropy index for each k in kv1 and kv2
necv	NEC index for each k in kv1 and kv2
lkv	log-likelihood at convergence of the EM algorithm for each k in kv1 and kv2
errv	trace of any errors occurred during the estimation process for each k in kv1 and kv2

Author(s)

Francesco Bartolucci, Silvia Bacci - University of Perugia (IT)

References

Bartolucci, F., Bacci, S. and Gnaldi, M. (2014), MultiLCIRT: An R package for multidimensional latent class item response models, *Computational Statistics & Data Analysis*, **71**, 971-985.

Examples

```
## Not run:
# Fit the model under different within-item multidimensional structures
# for SF12_nomiss data
data(SF12_nomiss)
S = SF12_nomiss[,1:12]
X = SF12_nomiss[,13]

# Partial credit model with two latent variables sharing six items
# (free difficulty parameters and constrained discriminating parameters;
# 1 to 3 latent classes for the 1st latent variable and 1 to 2 classes for the 2nd latent variable;
# one covariate):
multi1 = c(1:5, 8:12)
multi2 = c(6:12, 1)
out1 = search.model_within(S=S,kv1=1:3,kv2=1:2,X=X,link="global",disc=FALSE,
                           multi1=multi1,multi2=multi2,disp=TRUE,
                           out_se=TRUE,tol1=10^-4, tol2=10^-7, nrep=1)

# Main output
out1$lkv
out1$aicv
out1$bicv
# Model with 2 latent classes for each latent variable
out1$out.single[[4]]$k1
out1$out.single[[4]]$k2
```

```
out1$out.single[[4]]$Th1
out1$out.single[[4]]$Th2
out1$out.single[[4]]$piv1
out1$out.single[[4]]$piv2
out1$out.single[[4]]$ga1c
out1$out.single[[4]]$ga2c
out1$out.single[[4]]$Bec

## End(Not run)
```

SF12 *SF12 dataset*

Description

This data set contains the responses of 620 oncological patients to 12 ordinal polytomous items that measure the health-related quality of life, according to the Italian release of Short-Form 12 version 2 (SF-12v2); patient's age is also provided.

Usage

```
data(SF12)
```

Format

A dataframe with 620 observations on 12 items and one covariate:

`Y1` general health

`Y2` limits in moderate activities

`Y3` limits in climbing several flights of stairs

`Y4` accomplished less than he/she would like, as a result of his/her physical health

`Y5` limited in the kind of work or daily activities, as a result of his/her physical health

`Y6` accomplished less than he/she would like, as a result of his/her emotional health

`Y7` did work less carefully than usual, as a result of his/her emotional health

`Y8` how much did pain interfere with normal work

`Y9` how much of the time have he/she felt calm and peaceful

`Y10` how much of the time did he/she have a lot of energy

`Y11` how much of the time have he/she felt downhearted and depressed

`Y12` how much of the time physical health or emotional health interfered with social activities

`age` age of the respondent

Details

All items have 5 response categories, with the exception of items Y2 and Y3 having 3 response categories: the minimum value 0 correspond to a low level of quality of life, whereas the maximum value corresponds to a high level of quality of life. A proportion of 0.205 patients (127 out of 620) has missing responses (NA) on one or more items.

References

Ware, J., Kosinski, M., Turner-Bowker, D. and Gandek, B. (2002), *SF-12v2. How to score version 2 of the SF-12 health survey*, QualityMetric Incorporated: Lincoln.

Examples

```
data(SF12)
dim(SF12)
## maybe str(SF12)
str(SF12)
```

SF12_nomiss *SF12 dataset without missing responses*

Description

This data set contains the responses of 493 oncological patients to 12 ordinal polytomous items that measure the health-related quality of life, according to the Italian release of Short-Form 12 version 2 (SF-12v2); patient's age is also provided.

Usage

```
data(SF12)
```

Format

A dataframe with 493 observations on 12 items and one covariate:

- `Y1` general health
- `Y2` limits in moderate activities
- `Y3` limits in climbing several flights of stairs
- `Y4` accomplished less than he/she would like, as a result of his/her physical health
- `Y5` limited in the kind of work or daily activities, as a result of his/her physical health
- `Y6` accomplished less than he/she would like, as a result of his/her emotional health
- `Y7` did work less carefully than usual, as a result of his/her emotional health
- `Y8` how much did pain interfere with normal work
- `Y9` how much of the time have he/she felt calm and peaceful

`Y10` how much of the time did he/she have a lot of energy

`Y11` how much of the time have he/she felt downhearted and depressed

`Y12` how much of the time physical health or emotional health interfered with social activities

`age` age of the respondent

Details

All items have 5 response categories, with the exception of items Y2 and Y3 having 3 response categories: the minimum value 0 correspond to a low level of quality of life, whereas the maximum value corresponds to a high level of quality of life. All records are complete.

References

Ware, J., Kosinski, M., Turner-Bowker, D. and Gandek, B. (2002), *SF-12v2. How to score version 2 of the SF-12 health survey*, QualityMetric Incorporated: Lincoln.

Examples

```
data(SF12_nomiss)
dim(SF12_nomiss)
## maybe str(SF12_nomiss)
str(SF12_nomiss)
```

---

summary.est_multi_poly_between

Print the output of est_multi_poly_between object

Description

Given the output from est_multi_poly_between, it is written in a readable form

Usage

```
## S3 method for class 'est_multi_poly_between'
summary(object, ...)
```

Arguments

`object`	output from est_multi_poly_between
`...`	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

summary.est_multi_poly_within *Print the output of est_multi_poly_within object*

Description

Given the output from est_multi_poly_within, it is written in a readable form

Usage

```
 ## S3 method for class 'est_multi_poly_within'
summary(object, ...)
```

Arguments

object	output from est_multi_poly_within
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

vcov.est_multi_poly_between *Display the estimated variance-and-covariance matrix of est_multi_poly_between object*

Description

Given the output from est_multi_poly_between, the estimated variance-and-covariance matrix is displayed

Usage

```
 ## S3 method for class 'est_multi_poly_between'
vcov(object, ...)
```

Arguments

object	output from est_multi_poly_between
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

vcov.est_multi_poly_within *Display the estimated variance-and-covariance matrix of est_multi_poly_within object*

Description

Given the output from est_multi_poly_within, the estimated variance-and-covariance matrix is displayed

Usage

```
 ## S3 method for class 'est_multi_poly_within'
vcov(object, ...)
```

Arguments

object	output from est_multi_poly_within
...	further arguments passed to or from other methods

Author(s)

Francesco Bartolucci - University of Perugia (IT)

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